Isogeometric Discontinuous Galerkin method with deformable domains

Stefano Pezzano

Université Côte d'Azur, Inria, CNRS, LJAD e-mail: stefano.pezzano@inria.fr

Prix Pierre Laffitte, 23 October 2020

Work done in collaboration with Régis Duvigneau

- Methodology and results
- 3 Future perspective

- Methodology and results
- Future perspective

Flows with evolving geometries

• Aerospace Engineering









• Energy generation

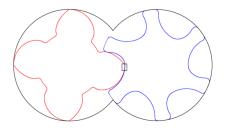
• Civil Engineering





IsoGeometric Analysis



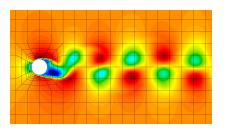


- Different representations used for design and analysis
- Expensive conversion from CAD to CFD format
- IsoGeometric Analysis: CAD description used in numerical methods

Discontinuous Galerkin methods

- High accuracy solvers are needed
- Uses polynomials of degree $p \ge 2$
- Allows very coarse meshes...
- ...if geometry description is accurate





6/17

- Isogeometric DG¹: employs a CAD-consistent representation
- Open-source implementation in the **Igloo** suite

S. Pezzano CAD-consistent ALE-DG Prix Laffitte 2020

¹R. Duvigneau, Isogeometric analysis for compressible flows using a Discontinuous Galerkin method, Comput. Methods Appl. Mech. Engrg. 333 (2018), 443-461

- Methodology and results
- Future perspective

ALE Methodology

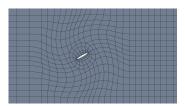
• Isogeometric DG, Eulerian:

$$\left(\int_{\widehat{\Omega}} R_k R_i \left| \mathsf{J}_{\Omega_j} \right| \mathsf{d}\widehat{\Omega} \right) \frac{\mathsf{d} \mathbf{w}_i}{\mathsf{d} t} = \int_{\widehat{\Omega}} \nabla R_k \cdot \mathbf{F} \left| \mathsf{J}_{\Omega_j} \right| \mathsf{d}\widehat{\Omega} - \oint_{\partial \widehat{\Omega}} R_k \, \mathbf{F}^* \left| \mathsf{J}_{\Gamma_j} \right| \mathsf{d}\widehat{\Gamma}$$

• Isogeometric DG, Arbitrary Lagrangian-Eulerian²:

$$\frac{\mathrm{d}}{\mathrm{d}t}\bigg(\mathbf{w}_i\int_{\widehat{\Omega}}R_kR_i\,|\mathsf{J}_{\Omega_j}|\,\mathrm{d}\widehat{\Omega}\bigg) = \int_{\widehat{\Omega}}\nabla R_k\cdot\big(\mathbf{F}-\mathbf{V}_g\mathbf{w}_h\big)\,|\mathsf{J}_{\Omega_j}|\,\mathrm{d}\widehat{\Omega} - \oint_{\partial\widehat{\Omega}}R_k\,\mathbf{F}_{ale}^*\,|\mathsf{J}_{\Gamma_j}|\,\mathrm{d}\widehat{\Gamma}$$

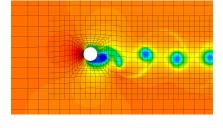
- Mesh movement algorithm
- Unified mathematical description

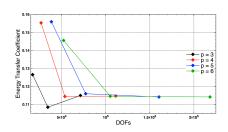


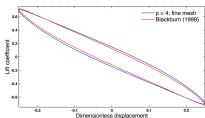
²Pezzano, Duvigneau, A NURBS-based Discontinuous Galerkin method for conservation laws with high-order moving meshes, under review, 2020

Oscillating Cylinder

- Simulation of Lock-in phenomenon
- Faster convergence with $p \ge 4$
- Good agreement with references

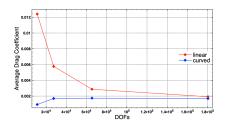


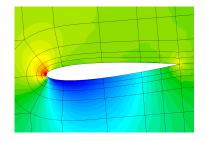


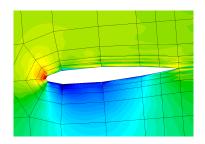


Influence of the geometry

- Oscillating airfoil flow
- Curvilinear vs rectilinear grid
- Non-physical effects with linear grid

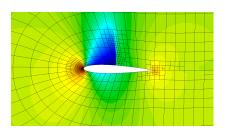


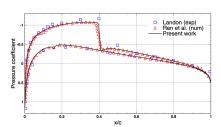




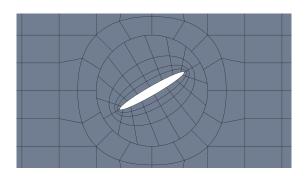
Adaptive Mesh Refinement

- Very coarse, but CAD-consistent, initial mesh
- Automatic refinement based on error indicator
- High mesh resolution only where needed
- Nearly user-independent simulation





Sliding Meshes



- Rotating machinery requires different mesh movement technique
- Geometrically exact sliding movement
- Fully conservative numerical scheme

Oscillating Ellipse

- Complex flow configuration
- Comparison with classic ALE
- Same results with sliding and deformation

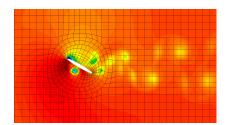
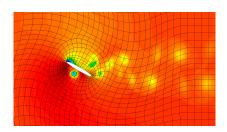


Table: Energy transfer coefficient

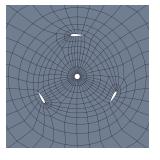
DOFs	Sliding	Deformation
13176	-0.7207	-0.7204
21654	-0.7070	-0.7059
45414	-0.7055	-0.7054



- Methodology and results
- 3 Future perspective

Application: Vertical Axis Wind Turbine

- Modelling and simulation of VAWT
- Coupling with shape optimization algorithm
- Fully integrated design chain prototype





Conclusion and perspectives

Current status:

- Successful development of a proof-of-concept solver, but...
- ...more powerful geometry manipulation tools are needed

Research perspective:

- Enhance integration with CAD platforms
- Extension to multi-physics problems

Industrial perspective:

- Tighter coupling between CFD and geometry is needed³
- Traditional CFD approaches are becoming obsolete

Efficient design methods represent the foundation of innovative products.

³NASA, CFD Vision 2030 Study: A Path to Revolutionary Computational Aerosciences

S. Pezzano CAD-consistent ALE-DG Prix Laffitte 2020

16 / 17

Thanks for your attention!

- e-mail: stefano.pezzano@inria.fr
- Personal page: https://stefanopezzano.netlify.app
- Gitlab repository: https://gitlab.inria.fr/igloo/igloo

